Application of Continuum Damage Theory for Flexural Fatigue Tests

Eng. Luiz Guilherme Rodrigues de Mello

National Department of Infrastructure on Transportation
NUMBERS from BRAZIL`s HIGHWAYS

Federal Highways (118.000km)

- Paved: 43,116.0 (36%)
- Unpaved: 13,772.0 (12%)
- Planned: 61,807.0 (52%)

Federal Highways Condition Evaluation


DNIT has approximately $4 billions for 2009/2010
INTRODUCTION

\[ N_f = K_1 \left( \frac{1}{\varepsilon} \right)^{K_2} \]

\[ \frac{da}{dN} = A \Delta K^n \]

\[ \dot{D}_m = \left( - \frac{\partial W^R}{\partial D_m} \right)^{\alpha_m} \]
INTRODUCTION

Works published on the subject:

- Park et al. (1996). A Viscoelastic Continuum Damage Model and its Application to Uniaxial Behavior of Asphalt Concrete;
- Daniel (2001). Development of a Simplified Fatigue Test and Analysis Procedure Using a Viscoelastic Continuum Damage Model and its Implementation to WestTrack Mixtures;
- Christensen & Bonaquist (2005). Practical Application of Continuum Damage Theory to Fatigue Phenomena in Asphalt Concrete Mixtures
- Baek et al. (2008). Viscoelastic Continuum Damage Model Based Finite Element Analysis of Fatigue Cracking;
"(…) The growth of short crack is then described in global way, using D instead of the crack length measure. The fact that short crack propagation does not follow the long crack behavior is a good justification to use a more global parametrization, in the framework of CD instead of Fracture Mechanics." Chaboche & Lesne (1988)

"(…) The evidence relating to the influence of microcracks on the mechanical response of solids over a wide spectrum of circumstances is too obvious to be neglected.” Krajcinovic (1989)

“(…) Description of damage evolution in materials using micromechanical analysis is in principle very difficult as the character of the microstructure and interaction among defects is difficult to characterize based on details of the micro-cracks. Due to this difficulty, CD mechanics models often ignore the physical details of the defects and instead simply focus on macroscopic response (e.g. stiffness).” Lundstrom et al. (2007)
CONTINUUM DAMAGE MECHANICS

Three aspects of the CDM theory are associated with the selection of:

✓ A “proper” mathematical representation for the damage variable;

✓ Particular objective form for the strain energy density (Green’s hyperelastic constitutive model);

✓ Appropriate form of the damage evolution laws;
CONTINUUM DAMAGE MECHANICS

Classical Continuum Damage

Scalar damage parameter: will never reaches the value $D = 1$. In this case we need a damage law (limit at $D = 0.5$ for example).

$$D = \frac{A - A_D}{A}$$

But sometimes damage and microcracks distribution might appear to be different!
CONTINUUM DAMAGE MECHANICS

Microcracks in planes perpendicular to the tensile axis:

- Sample will behave as damaged

If the sign of stress is reversed

- Sample will behave as undamaged

Krajcinovic, 1989
Effective stress concept

\[
E = \frac{\tilde{\sigma}}{\varepsilon} = \frac{\sigma}{(1-D)\varepsilon}
\]

\[
D = 1 - \frac{\tilde{E}}{E}
\]

The concept of scalar damage has some advantages as the constitutive equations for the damaged material are equivalent to those for the undamaged material, using the effective stress instead of the nominal stress. (Krajcinovic 1989)
CONTINUUM DAMAGE MECHANICS

CDM applied to viscoelastic materials

✓ Correspondence principle

✓ Work Potential Theory – WPT;

✓ Damage evolution law;

Schapery (1984, 1990); Park et al. (1996)
\[ \sigma(t) = \int_{0}^{t} E(t - \tau) \frac{\partial \varepsilon}{\partial \tau} d\tau \]

Schapery's extended correspondence principle suggest that elastic and viscoelastic constitutive equations are identical, but stress and strains in the viscoelastic body not necessarily are physical quantities but pseudo-elastic variables.

\[ \varepsilon^R(t) = \frac{1}{E_R} \int_{0}^{t} E(t - \tau) \frac{\partial \varepsilon}{\partial \tau} d\tau \]

\[ \varepsilon^R(t) = \frac{1}{E_R} \sigma(t) \]

The physical meaning of the pseudostrain \( (\varepsilon^R) \) corresponds to the linear viscoelastic stress for a given strain history.
CONTINUUM DAMAGE MECHANICS

How determine pseudostrain (sinusoidal loading)?

\[ \varepsilon^R = \frac{1}{E_R} \left[ \varepsilon'_0 |E^*| \sin(\omega t + \theta + \varphi) \right] \]

\[ \varepsilon^R = \frac{1}{E_R} \left[ \varepsilon'_0 |S^*| \sin(\omega t + \theta + \varphi) \right] \]

Uniaxial condition (Lee 1996)  Flexural condition

The pseudostrain concepts allow definition of the concept of pseudostiffness (C), which is the loss off stiffness solely due to loss of material integrity caused by damage and defined as follows:

\[ \sigma = C\varepsilon^R \]

Park et al (1996)
Work Potential Theory – WPT

\[ \sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} \]

Green's hyperelastic model

\[ W^R = f(\varepsilon^R, D) \]

Lee (1996) presented the pseudo-strain energy density function as the follow:

\[ W^R = C_0 \varepsilon^R + \frac{1}{2} C_1 (\varepsilon^R)^2 \]

\[ \sigma = C \varepsilon^R \]
**CONTINUUM DAMAGE MECHANICS**

✓ *Damage Evolution law*

\[
\dot{D}_m = \left(-\frac{\partial W^R}{\partial D_m}\right)^\alpha_m
\]

- \(\alpha\) is a material constant;
- Initially, it could be defined as viscoelastic property: relaxation or creep test;

**Numerical integration**

\[
D = \sum_{i=1}^{N} \left[ \frac{I}{2} \left( \varepsilon_i^R \right)^2 (C_{i-1} - C_i) \right]^{\alpha/(1+\alpha)} (t_i - t_{i-1})^{1/(1+\alpha)}
\]
CONTINUUM DAMAGE MECHANICS

Characteristic Curve $C \times D$

$$C = C_0 - C_1 \cdot D^{C_2}$$

$$C = \exp(-C_1 \cdot D^{C_2})$$

Relationship that define the damage evolution for a particular material/test/Specimen (!)
CONTINUUM DAMAGE MECHANICS

Daniel & Kim 2002

Lundström & Isacsson 2003

Damage parameter - D
Pseudo-stiffness - C

0 50000 100000 150000 200000 250000 300000

0 0.2 0.4 0.6 0.8 1

50/60
70/100
160/220

Daniel & Kim 2002
Lundström & Isacsson 2003
# MATERIALS AND METHODS

<table>
<thead>
<tr>
<th>Layer</th>
<th>% binder</th>
<th>% Voids</th>
<th>thickness(cm)</th>
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</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>5,5%</td>
<td>7%</td>
<td>variable</td>
</tr>
<tr>
<td>Gap graded</td>
<td>7%</td>
<td>9%</td>
<td>5,0</td>
</tr>
<tr>
<td>Open graded</td>
<td>9%</td>
<td>18%</td>
<td>1,3</td>
</tr>
</tbody>
</table>

![Layer Diagram]

- 1,30 cm – open graded
- 38,0 cm - Base
- Subgrade
- 5,0 cm – gap graded
- Variable conventional (Dense)
MATERIALS AND METHODS
Beams for flexural fatigue tests

- Mixtures were taken from construction sites;
- Molds were carefully filled with AC (heterogeneity);
- Compaction process using haversine loading (2Hz and 2.8 MPa): close to field observation;
- Approximately 250 fatigue tests.
MATERIALS AND METHODS

Height variation

Stiffness variation

Height variation
MATERIALS AND METHODS
# MATERIALS AND METHODS

<table>
<thead>
<tr>
<th>Projects</th>
<th>Mixture type</th>
<th>Loading mode</th>
<th>Test temperature</th>
<th>Tests</th>
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<tr>
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<td>21°C</td>
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<tr>
<td>TG4</td>
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<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
MATERIALS AND METHODS

Pronk & Erkens (2001)

![Graphs showing material properties and cycles](image)

**Wholer curves – 50% So**

- Sinal Haversine: $y = 0.002x^{0.16}$, $R^2 = 0.88$
- Sinal Senoidal: $y = 0.001x^{0.12}$, $R^2 = 0.93$

**Wholer curves – Wn/wn**

- Sinal Haversine: $y = 0.002x^{0.17}$, $R^2 = 0.82$
- Sinal Senoidal: $y = 0.001x^{0.10}$, $R^2 = 0.60$
MATERIALS AND METHODS

In summary the following conditions were used:

- **Air voids:** 7% for the conventional specimens, 9% for the gap graded specimens and 18% for the open graded specimens.

- **Load condition:** Constant strain level, 6 – 10 levels of the range (300-1900 microstrain)

- **Load frequency:** 10 Hz (2 and 5 Hz for validation)

- **Test temperature:** 4.4, 21, and 37.8°C for conventional and gap graded mixtures; 21.1, and 37.8°C for open graded mixtures.
MATERIALS AND METHODS

Fatigue tests (flexural) → Pseudo Strain calculation

Laboratory Data (ε, σ, N) → Pseudo stiffness calculation

Pseudo Strain calculation

\[ \varepsilon^p = \frac{1}{E_r} \int_0^t E(t-\tau) \frac{\partial \varepsilon}{\partial \tau} d\tau \]

Pseudo stiffness calculation

\[ C = \frac{\sigma}{\varepsilon^p} \]

Definition of parameter α

Damage Parameter

\[ D = \sum_{i=1}^{N} \left[ \frac{I}{2} (\varepsilon_i^p)^2 (C_{i+1} - C_i) \right]^{\alpha/(1+\alpha)} \left( \frac{t_i - t_{i-1}}{2} \right)^{1/(1+\alpha)} \]

Is C(D) unique?

No → Is C(D) unique?

Yes → END

Plot characteristic curve C(D)
RESULTS AND DISCUSSIONS

Classical results:
RESULTS AND DISCUSSIONS

10th
50th
500th
5000th
20000th
90000th

Dissipated energy
RESULTS AND DISCUSSIONS
RESULTS AND DISCUSSIONS

$k_2$ VARIATION WITH TEMPERATURE FOR CONVENTIONAL MIXTURES

\[ k_2(T) = k_2(70^\circ F) \cdot [1 - 0.001 \cdot (T - 70^\circ F)] \]

(Rauhut & Kennedy 1982)
Despite the variability founded and the amount of tests, $k_2$ will be less temperature susceptible for asphalt rubber mixtures (my point of view!)
RESULTS AND DISCUSSIONS

![Graph showing results and discussions for Kohls Ranch 100°F, 70°F, and 40°F.](image)

**Kohls Ranch 100°F**

\[ N_f = 4.41 \times 10^{-4.09} \]
\[ R^2 = 0.93 \]

**Kohls Ranch 70°F**

\[ N_f = 1.06 \times 10^{-4.62} \]
\[ R^2 = 0.91 \]

**Kohls Ranch 40°F**

\[ N_f = 5.12 \times 10^{-5.91} \]
\[ R^2 = 0.77 \]
RESULTS AND DISCUSSIONS

Failure criteria!!

(Lundstrom et al. 2004)
RESULTS AND DISCUSSIONS

![Graph showing stress-strain relationship for different N values.](image)

-N = 10
-N = 20
-N = 30
-N = 40

Stress (kPa)

Strain ($10^{-6}$ m/m)

Pseudo strain $\varepsilon^R$
RESULTS AND DISCUSSIONS
RESULTS AND DISCUSSIONS

Damage calculation

Daniel (2001) showed that in the case of cyclic loadings damage can only accumulate during the tensile loading portion of each cycle. Therefore, damage was calculated using only \(\frac{1}{4}\) of the entire loading time, which was the approximate period for tensile stresses under haversine loading (uniaxial tests):

\[
D_1 \approx \sum_{i=1}^{N} \left[ \frac{I}{2} \left( \varepsilon_m^R \right)^2 \cdot (C_{i-1} - C_i) \right]^{\frac{a}{1+a}} \cdot \left( \frac{t_i - t_{i-1}}{4} \right)^{\frac{1}{(1+a)}}
\]
RESULTS AND DISCUSSIONS

For flexural conditions, the time should be different:

\[
D_1 \approx \sum_{i=1}^{N} \left[ \frac{I}{2} \left( \varepsilon_m R \right)^2 \left( C_{i-1} - C_i \right) \right]^\frac{\alpha}{1+\alpha} \cdot \frac{t_i - t_{i-1}}{2} \left( \frac{1}{1+\alpha} \right)
\]
RESULTS AND DISCUSSIONS
RESULTS AND DISCUSSIONS

\[ C = C_0 - C_1 \cdot D^{C_2} \]

\[ \alpha = 2.36 \]

\[ T = 21^\circ C \]
RESULTS AND DISCUSSIONS

Damage parameter (D)

T = 37°C

T = 4°C

Damage parameter (D)
RESULTS AND DISCUSSIONS

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^\circ C$</td>
<td>0.99</td>
<td>1.11E-05</td>
<td>0.84</td>
<td>3.17</td>
</tr>
<tr>
<td>$21^\circ C$</td>
<td>1.03</td>
<td>2.39E-03</td>
<td>0.49</td>
<td>2.36</td>
</tr>
<tr>
<td>$37^\circ C$</td>
<td>1.03</td>
<td>1.02E-02</td>
<td>0.41</td>
<td>1.79</td>
</tr>
</tbody>
</table>

Also $k_2$ is lower for higher temperature!
RESULTS AND DISCUSSIONS

Comparison should be made using a pavement structural analysis!
RESULTS AND DISCUSSIONS

Open graded mix at T = 21°C

Gap graded mix at T = 5°C
RESULTS AND DISCUSSIONS

[Graph showing pseudo stiffness (C) vs. Damage parameter with flexural stiffness (MPa) values for different models.]
Comparison should be made using a pavement structural analysis!
RESULTS AND DISCUSSIONS

Characteristic curve should be independent of frequency loading:
RESULTS AND DISCUSSIONS

Why not temperature comparison for characteristic curves?

Daniel (2001) and Lundstrom et al. (2003) showed characteristic curve is also temperature independent. In my opinion, parameter $\alpha$ (also $k_2$) will be temperature dependent and a unique C x D curve will be able for different temperatures.
RESULTS AND DISCUSSIONS

Characteristic curve should be independent of loading mode:

![Graph showing pseudo stiffness (C) versus damage parameter (D) for strain-controlled and stress-controlled models. The graphs indicate that the characteristic curve is independent of loading mode.](image)
RESULTS AND DISCUSSIONS

As illustrated by different works (Lee et al. 2003; Kim et al. 2006), parameter $\alpha$ is directly related with coefficient as follows:

$$k_2 = 2.\alpha$$
RESULTS AND DISCUSSIONS

Prediction of fatigue life:

\[
N_f = \frac{f . (D_f)^{1+(1-C_2)\alpha}}{[1+(1-C_2)\alpha] . (0.5I_1C_1C_2)^\alpha} \cdot |S^*|^{-2\alpha} \cdot (\varepsilon_0)^{-2\alpha}
\]
RESULTS AND DISCUSSIONS

As we mention before, comparison among different mixtures should be done by pavement structural analysis. This can be done using numerical analysis. For example VECD+FEP software (Dr. Richard Kim):

![Diagram of pavement layers](image)

- **Conventional 15.24 cm**
- **Base 38.1 cm**
- **Subgrade**

![Diagram of pavement layers](image)

- **Gap graded 6.35 cm**
- **Conventional 7.60 cm**
- **Base 38.1 cm**
- **Subgrade**

open graded 1.30 cm
RESULTS AND DISCUSSIONS

Preliminary results:

\[ N = 5.0 \times 10^7 \]
WHAT DO WE HAVE RIGHT NOW

✓ VEPCD: ViscoElasticPlastic Continuum Damage

\[ \varepsilon = \varepsilon_{ve} + \varepsilon_{vp} = E_R \int_0^\xi D(\xi - \xi') \frac{d \left( \frac{\sigma}{C(S^*)} \right)}{d\xi'} d\xi' + \left( \frac{p+1}{C} \right)^{\frac{1}{p+1}} \left( \int_0^\xi \sigma^q d\xi \right)^{\frac{1}{p+1}} \]

(Chehab 2002)

✓ Probably some interaction between Continuum Damage and Fracture Mechanics
CONCLUSIONS

- The evolution of internal damage in hot mix asphalt (HMA) can be properly evaluated using the framework of the Continuum Damage Theory to determine its characteristic curve.

- The characteristic curves proved to be unique for a wide range of imposed strain amplitudes for both conventional and asphalt-rubber mixes.

- Tests with different temperatures, however, were better fitted by adopting different values of parameter $a$ in the damage evolution relation. This parameter reduces with the increase in temperature.

- The results of this research, using bending fatigue tests, corroborate the findings of other researchers about the uniqueness of the characteristic curve obtained from uniaxial fatigue tests subjected to direct tension.
CONCLUSIONS

- Despite the fact that some mechanical properties are indirectly obtained from bending tests, these are simpler to perform and are available and well known in several research centers.

- The uniqueness of the characteristic curve is an auspicious fact since it provides a means to characterize a HMA with fewer laboratory tests than other approaches. It also implies that such curves can be implemented in numerical codes to simulate the behavior of flexible pavements subject to a wide range of in field load conditions.
ACKNOWLEDGEMENTS