Investigation into the Size effect on Four Point Bending Fatigue Tests

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Presented to:
the 3rd 4PBB Conference
Introduction

Fatigue cracking

Fatigue damage
Rutting
Raveling
Others

Failure modes on pavement

Fatigue cracking
Introduction

Fatigue tests in the lab

Simple Flexure:
- 2-point bending
- 4-point bending
- 3-point bending
- Rotating cantilever

Direct Axial Loading:
- Uniaxial tension and compression with Cylindrical specimen
- Necked-cylindrical specimen

Diametral Loading:
- Indirect tensile test
Introduction

4PB test equipments
Introduction

4PB test equipments

RILEM committee TC 182 PEB

\[ N_{f,50} = a \cdot \varepsilon^{-\beta} \]

\begin{align*}
\text{T/C: France, Sweden} \\
\text{2PB: France, Belgium} \\
\text{3PB: the Netherlands} \\
\text{4PB: the Nethlands, UK, Poland, Portugal} \\
\text{ITT: Sweden}
\end{align*}

\[ N_{3PB} > N_{4PB} > N_{2PB} > N_{T/C} > N_{ITT} \]
Introduction

Stress-strain distribution

Inhomogeneous test

Homogenous test

Material property

Specimen property
Introduction

Influence factors for stress-strain field
Partial Healing model

**Theory**

[A.C. Pronk 2001]

The total dissipated energy $W_{tot}$ can be expressed as:

$$W_{tot} = W_{syst} + W_{fat} + W_{dis}$$

- **System loss $W_{syst}$**: can be ignored for good equipment.
- **Fatigue consumption $W_{fat}$**: decrease the stiffness and increase the phase angle.
  $$W_{fat} = \delta \cdot W_{dis}$$
- **Visco-elastic loss $W_{dis}$**: transformed into heat.
  $$W_{dis} = \pi \cdot \varepsilon \cdot \sigma \cdot \sin \phi$$

Stress vs. Strain:

- Loading
- Unloading

Diagram illustrating the relationship between stress and strain during loading and unloading.
Partial Healing model

Stiffness damage part $Q$

$$W_{fat} = \delta \cdot W_{dis} = \delta \cdot \pi \cdot \varepsilon \cdot \sigma \cdot \sin \phi = \delta \cdot \pi \cdot \varepsilon_0^2 \cdot S \cdot \sin \phi$$

$$\frac{d}{dt} Q = \frac{d}{dt} \delta \cdot W_{dis} \approx \delta \frac{\Delta W_{dis}}{\Delta t} = \delta \cdot f \cdot \pi \cdot \varepsilon_0^2 \cdot S \cdot \sin \phi$$

Describe the evolution of the loss and storage stiffness modulus during the fatigue test for a unit volume

Loss modulus

$$F\{t\} = S \cdot \sin \phi = F_0 - \int_0^t \frac{dQ\{\tau\}}{d\tau} \left[ \alpha_1 e^{-\beta(t-\tau)} + \gamma_1 \right] d\tau$$

Storage modulus

$$G\{t\} = S \cdot \cos \phi = G_0 - \int_0^t \frac{dQ\{\tau\}}{d\tau} \left[ \alpha_2 e^{-\beta(t-\tau)} + \gamma_2 \right] d\tau$$
Partial Healing model

Solutions for UT/C fatigue test

Loss modulus

\[ F\{t\} = S \cdot \sin \phi = F_0 e^{-Bt} \left[ Cosh\{Ct\} + DSinh\{Ct\} \right] \]

Storage modulus

\[ G\{t\} = G_0 - F_0 \left[ \frac{\alpha_2}{C} e^{-Bt} \cdot Sinh\{Ct\} + \frac{\gamma_2}{\gamma_1} \left( 1 - e^{-Bt} \cdot [Cosh\{Ct\} + ESinh\{Ct\}] \right) \right] \]

Model parameters: \( F_0, G_0, \alpha_1, \alpha_2, \beta^*, \gamma_1, \gamma_2 \)
Experimental work

Materials

Gradation of Dense asphalt concrete (DAC 0/8)

<table>
<thead>
<tr>
<th>Sieve size (mm)</th>
<th>0.063</th>
<th>0.18</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>2.8</th>
<th>4</th>
<th>5.6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt. %</td>
<td>11.2</td>
<td>19.6</td>
<td>21.5</td>
<td>33.2</td>
<td>7.9</td>
<td>6.5</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Composition of DAC 0/8

<table>
<thead>
<tr>
<th>Crushed stone</th>
<th>Crushed sand</th>
<th>Filler</th>
<th>Binder</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-5.6</td>
<td>5.6-4</td>
<td>6-2</td>
<td>2-0.18</td>
<td>0.18-0.063</td>
</tr>
<tr>
<td>Wt. %</td>
<td>11.2</td>
<td>19.6</td>
<td>21.5</td>
<td>33.2</td>
</tr>
</tbody>
</table>
**Experimental work**

### Dimensions of specimen

<table>
<thead>
<tr>
<th>Specimen size</th>
<th>Length [mm]</th>
<th>Width [mm]</th>
<th>Height [mm]</th>
<th>H/L ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>450</td>
<td>50</td>
<td>25</td>
<td>0.056</td>
</tr>
<tr>
<td>1.0</td>
<td>450</td>
<td>50</td>
<td>50</td>
<td>0.111</td>
</tr>
<tr>
<td>1.5</td>
<td>450</td>
<td>50</td>
<td>75</td>
<td>0.167</td>
</tr>
</tbody>
</table>
Experimental work

Test setup

Test conditions

<table>
<thead>
<tr>
<th>Loading mode</th>
<th>Strain-controlled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>20 °C</td>
</tr>
<tr>
<td>Frequency</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Waveform</td>
<td>sinusoidal</td>
</tr>
<tr>
<td>Strain level</td>
<td>50~200 μm/m</td>
</tr>
</tbody>
</table>

Size 0.5  Size 1.0  Size 1.5
**Test results and discussion**

**Initial stiffness**

![Graph showing initial stiffness vs. strain for different sizes](image)

\[ F_0 = S_0 \times \sin \varphi_0; \quad G_0 = S_0 \times \cos \varphi_0 \]
Evolution of stiffness and phase angle

Size 1.0, C-10-9, $\varepsilon_0 = 100$ $\mu$m/m

$\alpha_1=0; \; \alpha_2=937; \; \beta=68345; \; \gamma_1=21; \; \gamma_2=47$
Test results and discussion

Parameter $\alpha_1$, $\alpha_2$ & $\beta^*$

$$\alpha_1 = 0; \alpha_2 = a \cdot \varepsilon + b; \beta^* = \beta \cdot \varepsilon_0^2$$
Test results and discussion

**PH model parameters $\gamma_1$ & $\gamma_2$**

$\gamma_1 = \gamma_1^* \cdot (\varepsilon_0 - \varepsilon_{\text{limit}1})$; $\gamma_2 = \gamma_2^* \cdot (\varepsilon_0 - \varepsilon_{\text{limit}2})$

<table>
<thead>
<tr>
<th>Specimen size</th>
<th>Predicted endurance limit [$\mu$m/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>85~96</td>
</tr>
<tr>
<td>1.0</td>
<td>91~98</td>
</tr>
<tr>
<td>1.5</td>
<td>88~94</td>
</tr>
</tbody>
</table>
Test results and discussion

Verification

Model parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$F_0$</th>
<th>$G_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\beta^*$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>Relative error of $S$</th>
<th>Relative error of $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_0=155$ $\mu$m/m</td>
<td>3798</td>
<td>8152</td>
<td>0</td>
<td>1941</td>
<td>276</td>
<td>42</td>
<td>101</td>
<td>5.74%</td>
<td>5.28%</td>
</tr>
</tbody>
</table>

Relative error formula:

$$\text{Relative error} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{D_m - D_p}{D_m} \right| \times 100\%$$

- $D_m$: measured data
- $D_p$: predicted data
- $n$: total number of data
Test results and discussion

Fatigue life definition

- $N_{f,50}$: traditional fatigue life
- $N_R$: determined by dissipated energy ratio
- $N_{PH}$: determined by PH model

$$DER = \frac{\sum_{i=1}^{i=N} W_i}{w_N}$$
Test results and discussion

Comparison of fatigue life

<table>
<thead>
<tr>
<th>Size1</th>
<th>Strain level [μm/m]</th>
<th>NPH</th>
<th>NR</th>
<th>Nf,50</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>112</td>
<td>144</td>
<td>174</td>
<td>240</td>
</tr>
<tr>
<td>Number of cycle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000000</td>
<td>100000</td>
<td>10000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

17/09/2012
Test results and discussion

Size effect on fatigue life

\[ N_{PH} = k \cdot \varepsilon_0^b \]

<table>
<thead>
<tr>
<th>Specimen size</th>
<th>Material coefficient</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k</td>
<td>b</td>
</tr>
<tr>
<td>0.5</td>
<td>5.73E+14</td>
<td>-4.43</td>
</tr>
<tr>
<td>1.0</td>
<td>2.50E+14</td>
<td>-4.24</td>
</tr>
<tr>
<td>1.5</td>
<td>2.67E+14</td>
<td>-4.74</td>
</tr>
<tr>
<td>0.5+1+1.5</td>
<td>6.21E+14</td>
<td>-4.44</td>
</tr>
</tbody>
</table>
Conclusions

- PH model provides a good prediction for the evolution of complex stiffness modulus and phase angle in the uniaxial tension and compression test (UT/C test).

- The model parameter $\gamma_1$ and $\gamma_2$ can be used to determine the range of endurance limit, and this range does not change significantly with the increase of the specimen size. But more tests are needed to validate this.

- $N_{PH}$ and $N_R$ are close to each other compared to the traditional fatigue life $N_{f,50}$

- The fatigue life obtained from the UT/C fatigue test is independent of the specimen size.
Study in the future

- Validate of the endurance limit \( \varepsilon_{\text{limit}} \) predicted by the PH model

- Based on the UT/C test results, apply the PH model to the inhomogeneous fatigue tests, e.g. 4-point bending, 2-point bending fatigue tests, etc.

- Investigate the influence of temperature, loading mode.
Thanks for your attention!