Preface: This “paper” deals with a renewed interpretation of 4PB measurements carried out by Andre Cocurullo and presented at the RILEM conference in 2007. For the new interpretation the ACP-F model is used.
Introduction

It is well known that the fatigue life of a specimen can be enlarged if instead of continuous load applications, also rest periods are applied in a fatigue test. In this "report" a renewed interpretation is presented using the 4PB fatigue tests carried out by A. Cocurullo as a part of his MSc study in Delft and published at the RILEM conference in 2007. At that time, it was assumed that the influences of the outer sections in a 4PB test device could be neglected on the overall response of the specimen. However, recent developments make it clear that for the determination of the overall response of the beam (this is what will be measured) an integral model is needed taking into account the decrease of the complex stiffness modulus in the two outer sections and the inner midsection as well. The A(sphalt) C(oncrete) P(avement) – F(atigue) model (successor of the M(odified) P(artial) H(ealing) model is such an integral model and is used for the renewed interpretation of the test data.

ACP-F model

The ACP-F model is a model using a mesh of 10*10 elements for the upper (or bottom) half of one outer section and 1*10 elements for the upper half of the half inner section (midsection). These elements describe the complex stiffness modulus distribution in a quarter of the beam. Taking into account the (static) bending moments of these 110 elements the overall response for the beam can be calculated and is allowed for low frequencies up to 10 Hz. In the case of a controlled deflection (strain) controlled test the evolutions for the loss modulus $L$ and storage modulus $S$ can be analytical calculated for each element. For more detail the reader is referred to the papers of the 3rd international 4PB Workshop which are available on the 4PB platform of the university of Minho. The goal of the model is to fit both the calculated loss modulus $L_{\text{beam}}$ and the storage modulus $S_{\text{beam}}$ with the measured responses for the beam on the interval of $N_0$ to $N_1$ cycles. For the fitting the evolutions for the modulus $M_{\text{beam}}$ and phase lag $\phi_{\text{beam}}$ for the beam are also taken into account. The start of the fit interval $N_0$ (representing a time period of 5 minutes) is determined by the applied frequency $f$ [Hz]: $N_0 = 300$ times $f$. It is assumed that after 5 minutes a kind of equilibrium is reached for the temperature increase due to the dissipated energy per cycle. The decrease due to thixotropy is simulated by two parameters. The end value $N_1$ is defined as the number at which the measured dissipated energy ratio for the beam starts to deviate from a straight line through the origin. The goal function for the fit is the combined sum of the summed squared differences between the measured and calculated responses for $M_{\text{beam}}$, $L_{\text{beam}}$, $S_{\text{beam}}$ and $\phi_{\text{beam}}$. Each separate sum is weighted by dividing the sum by the variance of the measured responses on the fit interval.
Fit Protocol

The reader is referred to the Annex in part III for the outline of the analytical expressions for the ACP-F model. It is tricky to vary in one step all eight model parameters. Therefore, a kind of protocol is used in the fitting process:

1. The first step is the choice for the coefficient Power in the fatigue damage functions for the parameters $\gamma_1$ and $\gamma_2$. Normally the value is determined by the slope of the Wöhler curve for $N_1$ and $\varepsilon$. In this case only one strain value of 161 micro strain was used for all tests. Therefore, and based on earlier findings for a similar asphalt mixture the coefficient Power is taken equal to nil.

2. Because the decrease in modulus $M$ can be higher than the increase in the sine of the phase lag $\phi$, a negative value for $Tg\alpha_1$ is possible.

3. To avoid strange/odd evolutions for the calculated phase lags $\phi_{beam}$ and $\phi_{mat}$ (the last value is the phase lag at the surface in the midsection), three boundary conditions are set in the fitting procedure. 1) the ratio of $\phi_{mat}$ and $\phi_{beam}$ should be bigger than 1, 2) the slope of $\phi_{beam}$ must be positive and 3) the same requirement is taken for $\phi_{mat}$. These conditions should be used for at least on the interval from $N_0$ to $N_1$.

4. The second step is the choice of seed (initial) values for the parameters: - $Tg\alpha_1$ is given the value $-2.10^5$; $Tg\alpha_2$ is given the value $+2.10^6$ - $Tg\gamma_1$ is given the value $+2.10^6$; $Tg\gamma_2$ is given the value $+5.10^6$ - $\beta$ is taken equal to $+5.10^4$; $\varepsilon_{endurance}$ is taken equal to 50 $\mu$m/m.

5. The seed values for the initial Loss modulus $L_0$ and storage modulus $S_0$ are taken equal to the measured values for the weighted loss modulus and storage modulus of the beam at cycle 100.

6. The separate sum of the squared differences between the measured loss modulus $F$ for the beam and the calculated model value over the fit interval from $N = N_0$ (at the frequency of 5 Hz this is 1500) to $N = N_1$ is minimized by varying only the parameters $Tg\alpha_1$ and $Tg\gamma_1$.

7. The separate sum of the squared differences between the measured phase lag $\phi$ for the beam and the calculated model value over the fit interval from $N_0$ to $N_1$ is minimized by varying only the parameters $Tg\alpha_2$ and $Tg\gamma_2$.

8. In the following protocol steps the entire sum for the squared weighted differences (modulus $M_{beam}$, phase lag $\phi_{beam}$, loss modulus $L_{beam}$ and storage modulus $S_{beam}$) between the measured values and the model calculations for the response of the beam, is taken as the goal function for the minimization.

9. The goal function is minimized by only varying the parameter $\beta$.

10. The goal function is minimized by only varying the parameter $\varepsilon_{endurance}$.

11. The goal function is minimized by only varying the parameter $L_0$ and $S_0$.

12. Finally, the goal function is minimized by varying all parameters except $L_0$ and $S_0$ (initial). If necessarily it is possible to carry out step 9 again.
In this way a smooth iteration process will in general be obtained. The negative value 
for the $T_g\alpha_1$ seems odd but strange enough for the mix at issue the increase in the 
loss modulus for the beam due to the increase in the phase lag $\phi$ was less than the 
decrease in the modulus $M_{\text{beam}}$ of the beam. If the fatigue life $N_{\text{PH}}$ is much bigger than 
the N1 value, a new fit interval can be chosen with a bigger N1 value in order to 
check if the new $N_{\text{PH}}$ value deviates substantially from the former value.

A special remark with respect to mixtures with a polymer modified bitumen. For 
these mixes the initial drop in both the loss modulus and storage modulus is very big. 
To avoid non-realistic calculated evolutions for the material phase at the surface of 
the beam in the midsection a special requirement is added. At least on the interval 
from N0 to N1 the ratio of the material phase lag and the calculated phase lag for the 
beam should be bigger than 1. A second requirement is that the slopes of the 
evolutions for the material phase lag and the beam phase lag should be equal or 
bigger than zero.

**Part 1: Continuous tests (Beams 24-13, 25-04 & 26-01)**

In front of the discontinuous fatigue tests, three continuous fatigue tests in controlled 
deflection mode at an initial strain level of 160 micro strain were carried out. The 
goal of these three tests was to get an indication of the fatigue lives. In this way the 
number of loading cycles in one load period for the discontinuous test can be 
established at which around one third of the fatigue life was reached. The three beams 
were encoded as Beam 24-13, Beam 25-04 and Beam 26-01. In the following figures 
the results are given applying the ACP-F model.

**Beam 24-13**

The first step according to the protocol is the determination for the fitting interval 
from $N = N_0$ to $N = N_1$. The determination of N1 is given in figure 1. 
In view of the applied frequency of 5 Hz it is assumed that after $N_0 = 1500$ cycles (5 
minutes) the influence of the raise in temperature (due to the dissipated energy per 
cycle) is limited. Still both temperature and thixotropy play a role in the beginning 
(phase I). Later the slopes of the evolutions become nearly constant. The fatigue 
damage will play the dominant role and this part of the evolution is called the fatigue 
initiation phase (II).
Figure 1. Determination of N1 for beam 24-13

In figure 2 is the comparison given between the measured loss modulus for the beam and the fitted loss modulus according to the model. Mark that the comparison between measured and calculated evolution remains very good up to cycle 110,000.

Figure 2. Comparison of measured and calculated loss modulus $L_{beam}$ (N1=60,000).
Finally, the overall comparison after the fit procedure is given in figure 3. Notice the calculated fast increase in the (material) phase lag $\varphi_{\text{mat}}$ at the surface in the midsection of the beam and the decrease in the (material) modulus $M_{\text{mat}}$ at the surface in the midsection of the beam.

At last the fatigue life $N_{\text{ph}}$ is determined by comparing the measured and calculated dissipated energy ratio as function of the applied cycles. The comparison is given in figure 4 and a value of 112,000 cycles is adopted for the fatigue life $N_{\text{ph}}$. It can be argued that in view of the calculated evolutions for the loss modulus $L$ and modulus $M$ for the beam response (see figure 3) a value of $N_{\text{ph}} = 80,000$ cycles would be more appropriate. This last value is close to the traditional fatigue life $N_{f,50}$.

![Figure 3. Comparison of measured and calculated beam responses and the calculated material responses at the surface in the midsection ($N_1=60,000$).](image-url)
As mentioned before the determination of $N_{PH}$ is rather subjective to the judgement of the interpreter. Another definition is developed by Geoff Rowe (Abatech) which is more objective in the determination process. The determination is performed using a graph in which the product of cycle $N$ times the beam stiffness modulus $M_{beam}$ in that cycle is plotted as a function of the cycle number $N$. For strain/deflection controlled (4PB) fatigue tests this will lead to a monotonic increasing curve but with a decreasing slope. According to G. Rowe the number at which the slope becomes nil (maximum) is defined as the fatigue life ($N_{MN}$). The target was to obtain an objective value in the transition zone (TZ) between the fatigue initiation phase (II) and the failure phase (III). Besides the value $N$ also the square root of $N$ is used for the multiplication with the beam modulus $M_{beam}$ leading to a fatigue life $N_{MN}^*$. For beam 24-13 both curves are given in figure 5.
Figure 5. The product of $M_{\text{beam}}$ times cycle $N$ and $\sqrt{N}$ as a function of $N$.

If the maximum (slope = 0) of $N$ times $M_{\text{beam}}$ determines the fatigue life $N_{\text{MN}}$ than the value will be more than 140,000 cycles. The fatigue life $N_{f,50}$ is around 96,000 and the $N_{PH}$ value is around 112,000. The maximum of $N_{* \text{MN}}$ value is between 90,000 and 120,000 cycles. It should be noticed that in the evolution of $M_{\text{beam}}$ no real failure phase (III) is present (see figure 3).

Given the fact that the model calculation for the dissipated energy ratio follows the measured evolution very well above $N_1$, it was decided to perform a second fit procedure with a $N_1$ value of 100,000 cycles.

The obtained calculated evolutions are given in figures 6 to 8.

The calculated dissipated energy ratio follows the measured evolution up to an $N_{PH}$ value of 130,000 cycles. It can be argued that it is even more than 130,000 cycles.

Figure 6. Comparison of measured and calculated loss modulus $L_{\text{beam}}$ ($N_1=100,000$).
Figure 7. Comparison of measured and calculated beam responses and the calculated material responses at the surface in the midsection (N1=100,000)
Figure 8. Comparison of the measured and fitted dissipated energy ratios (N1=100,000).
Beam 25-04

For beam 25-04 a value of 60,000 cycles is adopted for the end of the fit interval (figure 9). At the end of the fit procedure the comparisons between measured and calculated beam responses are quite reasonable. However, after the end of the fit interval at N1, the calculated evolutions are a bit lower than the measured ones (figure 10 and 11). This is in contrast with normal findings. The opposite kind of deviation occurs at the plot of the dissipated energy ratios in figure 12. The fatigue life $N_{PH}$ is taken at $N = 118,000$ cycles but any value round 120,000 would be fine. Because there are doubts, the whole processing is carried out again with N1 is 100,000 cycles given the pre-assumption that the model should be correct up to N1 cycles. The results are graphical presented in figures 13 to 16.

Figure 9. Measured dissipated energy ratio versus N for beam 25-04.
Figure 9. Comparison of measured and calculated loss modulus for 25-04.

Figure 10. Comparison of measured and calculated beam responses and the calculated material responses at the surface in the midsection (N1=60,000).
Notice the decreasing evolution for the phase lag $\varphi_{beam}$ above 60,000 cycles (outside the fit interval).
Figure 11. Comparison of the measured and fitted dissipated energy ratios (N1=60,000).

![Figure 11](image1.png)

**Beam 25-04**

N_{PH} = 118,000

N1 = 60,000

Figure 12. The product of $M_{beam}$ times cycle N and $\sqrt{N}$ as a function of N

The alternative fatigue lives $N_{MN}$ and $N^*_{MN}$ are around 120,000 and 100,000 respectively. Looking at the measured evolutions in figure 10 the fatigue life should be in the neighbourhood of 120,000 cycles. Therefore, the fitting procedure is again carried out using N1 = 100,000 cycles as the end for the fit interval. The results are given in figures 13 to 16.
Figure 13. Measured dissipated energy ratio versus $N$ for beam 25-04.
- Loss Modulus from 4PB test
- Loss Modulus from ACP-F model

The end for the fit interval is $N = 100,000$

Figure 14. Comparison of measured and calculated loss modulus for 25-04 ($N1=100,000$).
Mark that the calculated loss modulus follows the measured evolution very well up to 130,000 cycles.

Figure 15. Comparison of measured and calculated beam responses and the calculated material responses at the surface in the midsection (N1=100,000).

Notice that the phase lag for the beam in figure 15 is still increasing in contrast with the evolution given in figure 10 (taking N1 = 60,000). Mark also the good comparison for the measured and calculated evolutions of $M_{beam}$. 
In contrast with the evolutions for beam 24-13 a failure phase (III) is clearly present in the measured evolutions for beam 25-04 (figure 15) starting around N = 128,000.

The second fit procedure using N1 = 100,000 has led to an overall better comparison between measured and calculated evolutions. The N_{PH} value is increased to 128,000 (see figure 16).

Figure 16. Comparison of the measured and fitted dissipated energy ratios (N1=100,000).
Beam 26-01

The interval for the first fitting was taken from 1,500 to 70,000 cycles (figure 17).

Figure 17. Measured dissipated energy ratio versus N for beam 26-01.

The result of the first fitting for the evolution of the beam loss modulus is given in figure 18. As shown the comparison remains very good up to 150,000 cycles. The results of the total comparison are given in figure 19. The three modulus evolutions remain very good up to 150,000 but the evolutions of the beam and material phase lags are less. Nevertheless, the calculated dissipated energy ratio follows the measured ratio up to an N_{PH} value even above 140,000 cycles (figure 20). The proposed method of M_{beam} times N leads also to a fatigue life above 140,000 cycles (figure 21). Again, a second fit was carried out from N_0=1,500 to N_1=110,000 cycles. These results are given in figures 22 to 26.
Figure 18. Comparison of measured and calculated loss modulus for 26-01 (N1=70,000).

Figure 19. Comparison of measured and calculated beam responses and the calculated material responses at the surface in the midsection (N1=70,000).
**Figure 20.** Comparison of the measured and fitted dissipated energy ratios (N1=70,000).

**Figure 21.** The product of $M_{\text{beam}}$ times cycle $N$ and $\sqrt{N}$ as a function of $N$

The method of $M_{\text{beam}}$ times $N$ and $\sqrt{N}$ lead also to higher fatigue lives just above 150,000 ($N_{MN}$) and around 120,000 ($N^{*}_{MN}$).

Looking at the measured evolutions in figure 19 shows that the fatigue failure phase (III) will be above 140,000 cycles.
Figure 22. Measured dissipated energy ratio versus N for beam 26-01.

Figure 23. Comparison of measured and calculated loss modulus for 26-01 using a fit interval from N0=3,000 to N1=110,000 cycles.

Mark that no fatigue failure phase (III) is present in the measured evolutions.
Figure 24. Comparison of measured and calculated beam responses and the calculated material responses at the surface in the midsection (N1=110,000).

Figure 25. Comparison of the measured and fitted dissipated energy ratios (N1=110,000).
Preliminary Conclusions with respect to the fatigue lives.

Based on these three continuous tests it can be concluded that the fatigue lives for this mix using a strain amplitude of 161 micro strain is 120,000 or more. Therefore, the following discontinuous tests were carried out with load periods of 40,000 cycles followed by rest periods of 400,000 cycles.

The $N_{PH}$, $N_{MN}$ and $N_{MN}^*$ values are close to each other.

Fitted values for the ACP-F parameters

Parameters describing the thixotropic behaviour

The back calculated values for the thixotropic behaviour of the beams (including thermal effects) during bending are given in table 1 for two N1 values. As can be seen the change is not so big when the fit interval is enlarged. However, the differences in parameter values for different beams are really big, especially for the $T_g\alpha_1$ value (from -0.72 to -25.6).

In a later note this difference will be discussed in detail.

Table 1. Fitted values for the thixotropic parameters in the continuous tests.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$T_g\alpha_1/10^5$ [%]</th>
<th>$T_g\alpha_2/10^6$ [%]</th>
<th>$\beta/10^4$ [1/s]</th>
<th>$\varepsilon_{endurance}$ [$\mu$m/m]</th>
<th>$N_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24-13</td>
<td>-1.29</td>
<td>-0.72</td>
<td>2.78</td>
<td>2.79</td>
<td>2.03</td>
</tr>
<tr>
<td>25-04</td>
<td>-23.9</td>
<td>-25.6</td>
<td>18.7</td>
<td>17.4</td>
<td>10.1</td>
</tr>
<tr>
<td>26-01</td>
<td>-15.3</td>
<td>-14.8</td>
<td>19.2</td>
<td>14.1</td>
<td>8.51</td>
</tr>
</tbody>
</table>

Parameters describing the fatigue damage behaviour

In table 2 the fitted values for the ACP-F fatigue damage parameters are given. Again, the changes in values due to a bigger fit interval are small. Fortunately, the variation in the parameter $T_g\gamma_1$ for different beams is also small. This parameter is most relevant for the evolution of the dissipated energy per cycle. In view of the values obtained for the endurance limit the increase of the fit interval from the original N1 value to a value close to $N_{PH}$ looks better.

Table 2. Fitted values for the fatigue damage parameters in the continuous tests.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$T_g\gamma_1/10^6$ [%]</th>
<th>$T_g\gamma_2/10^6$ [%]</th>
<th>$\varepsilon_{endurance}$ [$\mu$m/m]</th>
<th>$N_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24-13</td>
<td>1.84</td>
<td>1.83</td>
<td>3.15</td>
<td>61.9</td>
</tr>
<tr>
<td>25-04</td>
<td>2.11</td>
<td>2.41</td>
<td>3.47</td>
<td>35.0</td>
</tr>
<tr>
<td>26-01</td>
<td>1.52</td>
<td>1.53</td>
<td>2.73</td>
<td>45.9</td>
</tr>
</tbody>
</table>
Parameters for the fitted initial modulus on the fit interval and the modulus values at cycle 100.

In table 3 the values are given for the initial loss modulus L, storage modulus S and the modulus M calculated from the fitting of the ACP-F model on the fit interval. In table 4 the values are given following the standards which defines the initial modulus as the value measured in cycle 100

Initial values for the modulus

Table 3. Fitted values for the initial modulus parameters according to ACP-F.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Mo [MPa]</th>
<th>Lo [MPa]</th>
<th>So [MPa]</th>
<th>N1</th>
</tr>
</thead>
<tbody>
<tr>
<td>24-13</td>
<td>6821</td>
<td>6831</td>
<td>3526</td>
<td>3529</td>
</tr>
<tr>
<td>25-04</td>
<td>6863</td>
<td>6805</td>
<td>3569</td>
<td>3550</td>
</tr>
<tr>
<td>26-01</td>
<td>7242</td>
<td>7113</td>
<td>3635</td>
<td>3649</td>
</tr>
</tbody>
</table>

As shown in table 4 the initial modulus at cycle 100 is substantial higher than the fitted values given in table 3. This difference is probably due to the fast-initial decrease in modulus due to heating by the dissipated energy per cycle. It is assumed that after 5 minutes (1500 cycles) a kind of equilibrium is reached with a higher temperature.

Table 4. Initial modulus values as measured in cycle 100.

<table>
<thead>
<tr>
<th>Beam</th>
<th>M_{100} [MPa]</th>
<th>L_{100} [MPa]</th>
<th>S_{100} [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24-13</td>
<td>7473</td>
<td>3564</td>
<td>6569</td>
</tr>
<tr>
<td>25-04</td>
<td>7143</td>
<td>3649</td>
<td>6114</td>
</tr>
<tr>
<td>26-01</td>
<td>7387</td>
<td>3626</td>
<td>6435</td>
</tr>
</tbody>
</table>

Fatigue lives calculation according to different definitions

In table 5 the different values for the fatigue lives are given. It is quite clear that the common definition by N_{f,50} gives substantial lower values. The values according to the ACP-F model and N_{GR} are of the same order. In the set-up procedure of these tests it was decided to end the fatigue tests just after N_{f,50} was reached. Only for beam 25-04 the test was ended later (see figure 15).

Table 5. Calculated fatigue lives for the beams 24-13, 25-04 and 26-01

<table>
<thead>
<tr>
<th>Beam</th>
<th>N_{PH} [c]</th>
<th>N_{f,50} [c]</th>
<th>N_{MN} [c]</th>
<th>N1 [c]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24-13</td>
<td>110,000</td>
<td>130,000</td>
<td>&gt;140,000</td>
<td>60,000 ; 100,000</td>
</tr>
<tr>
<td>25-04</td>
<td>118,000</td>
<td>128000</td>
<td>120,000</td>
<td>60,000 ; 100,000</td>
</tr>
<tr>
<td>26-01</td>
<td>&gt;150,000</td>
<td>&gt;150,000</td>
<td>&gt;150,000</td>
<td>70,000 ; 110,000</td>
</tr>
</tbody>
</table>
**Applied Strain amplitude**

During the fatigue test the (weighed) stiffness modulus for the beam will decrease. This has an influence on the process control of the deflection amplitude itself and so on the strain amplitude, which is calculated from the deflection. The deflection amplitude is kept constant using a so called PID controller. In the following graphs graphical presentations are given for the evolutions of the strain amplitudes.

**Figure 26. Evolution of the strain amplitude for beam 24-13.**

**Figure 27. Evolution of the strain amplitude for beam 24-13 up to cycle 100.**
Figure 28. Evolution of the strain amplitude for beam 25-04.

Figure 29. Evolution of the strain amplitude for beam 25-04 up to cycle 100.

Figure 30. Evolution of the strain amplitude for beam 26-01.
As shown in the figures 26 to 31 the target value of 161 micro strain is reached within 20 cycles. The variation in the obtained strain amplitudes stays within +/- 2 micro strain over the whole measured interval up to 160,000 cycles.

**Conclusions**

- The fatigue lives $N_{PH}$, $N_{MN}$ and $N^*_{MN}$ are significantly higher than the traditional fatigue life $N_{f,50}$. In view of the absence of the failure phase (III) for beams 24-13 and 26-01 it can be concluded that at least for these two beams the fatigue life $N_{f,50}$ underestimates the “correct” fatigue life which will be in the order of 120,000 cycles.
- Based on the fact that the fatigue life is in the order of 120,000 cycles the load periods for the discontinuous 4PB tests are taken equal to 40,000 cycles. The rest period is taken equal to ten times the load period = 400,000 cycles.
- The strain amplitudes stay during the whole test within a range of 4 µm/m. The target strain amplitude of 161 µm/m is reached within 30 cycles.
- In view of the good comparison it might be an option to take a lower $N_0$. For the discontinuous tests the start value of the fit interval will be taken equal to 30 cycles. It is hoped that both the changes due to thixotropy and temperature can be simulated by only one equation.
- A second fitting procedure using a $N_1$ value bigger than the first value for $N_1$ which is obtained from a straight line through the dissipated energy ratio may lead to better comparisons between measured and calculated evolutions. Therefore, it is advised to carry out a second fit if $N_1$ is small compared to $N_{PH}$. 

Figure 31. Evolution of the strain amplitude for beam 26-01 up to cycle 100.